The electron and the holographic mass solution

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Abstract. A computation of the electron mass is found utilizing a generalized holographic mass solution in terms of quantum electromagnetic vacuum fluctuations. The solution gives a clear insight into the structure of the hydrogen Bohr atom, in terms of the electron cloud and its relationship to the proton and the Planck scale vacuum fluctuations. Our electron mass derivation is in agreement with the measured CODATA 2014 value. As a result, an elucidation of the source of the fine structure constant α, the Rydberg constant R_∞, and the proton-to-electron mass ratio μ is determined to be in terms of vacuum energy interacting at the Planck scale.

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1. Introduction

Measurements of the electron mass are typically determined utilizing penning traps, where the latest measured CODATA 2014 value is given as $9.10938356(11)\times10^{-28}g$ **[1]**. However, although measurements are extremely precise a satisfactory derivation from first principles has yet to be found and thus the nature of the electron remains a mystery.

The Bohr model considers the electron as an extended source while relativistic quantum field theory (QED) treats the electron and positron as point particles with no internal structure yet each possessing an intrinsic angular momentum, or spin. Furthermore, the point-like nature of the electron, in quantum field theories, leads to an infinite bare mass and bare charge. Therefore, to agree with measurement, the mass of the electron is subsequently given in terms of two infinities, the bare mass and the radiative corrections, renormalizing to a finite value [2] [3]. This value can be given in terms of fundamental constants, including the Rydberg constant, defining the standard mass of the electron as,

$$m_e = \frac{2R_\infty h}{c\alpha^2} = 9.10938356(11)\times10^{-28}g \qquad \text{Eqn. 1}$$

giving a value in agreement with the measured CODATA 2014 value. However, this standard derivation, in terms of the Rydberg constant, does not reveal the nature or structure of the electron, or give us insight into the source of mass and charge.

Currently, no experimental evidence supports the point-like view of the electron, although Crater and Wong suggest that such a view would be supported if the existence of a peculiar ground singlet state ${}^1S_0$ is found [4] or at the least could provide an experimental limit on the point-like nature. However, as noted by Frank Wilczek, *“An electron’s structure is revealed only when one supplies enough energy […] at least 1 MeV, which corresponds to the unearthly temperature of 10^{10} kelvin”* below which it ‘appears’ point-like and structure-less [5].

Although the position and momentum can only be defined in terms of a probability cloud, the quantum behavior of the electron is successfully calculated by the current standard model. Yet the most precise prediction being that of the g-factor [6] [7] where the observed deviation [8], known as the anomalous magnetic moment, still requires the inclusion of a contribution from quantum vacuum fluctuations [9]. Quantum corrections are also expected for an electric field – but as yet no such field has been detected. Based on charge-parity (CP) violating components the standard model assumes an upper limit on the electron electric dipole moment (EDM) of $d_e \leq 10^{-38} q$ cm, [10] which is smaller than current experimental sensitivities. However recent experiments confirm a non-zero EDM e.g. [11] and [12] who find $d_e < 10.5 \times 10^{-28} q$ cm and $d_e < 6.05 \times 10^{-25} q$ cm, respectively, suggesting the standard model is incomplete and there must be other sources of CP violation. Higher EDMs are predicted by extensions to the standard model e.g. supersymmetric models, which predict $d_e > 10^{-26} q$ cm [13].

Defining the fundamental characteristics of particles from first principles, and without free parameters, is of great importance as not only will it provide information about the structure of subatomic particles but also the source of mass and the nature of spacetime itself. Successful predictions allow us to confirm and improve upon existing models.

The current source of mass, according to the standard model, is through the interaction with the Higgs field, where as the 'mass terms' violate gauge symmetry, a measurable mass is only acquired through symmetry breaking. However, the Higgs mechanism introduces complexities with a non-zero vacuum expectation value which only predicts 1 to 5 percent of the mass of baryons, and in which the Higgs particle mass itself is a free parameter [14].

In earlier work a geometric model was proposed utilizing a generalised holographic mass solution, which successfully computed the mass of the proton [15]. As a result, a precise charge radius value was found which is within an 1σ agreement with the latest muonic measurements of the charge radius of the proton [16], relative to a 7σ variance in the standard approach [17]. Utilizing this model, we now extend our holographic mass solution in an attempt to deepen our understanding of the electron and its relationship with the Planck scale vacuum fluctuations. Specifically, our result computes the mass of the electron in terms of surface-to-volume ratios of Planck oscillator information bits with a value in agreement with the measured CODATA 2014 value.

This new definition, with a source for mass, successfully predicts the energy levels for the currently known quantum states of the Hydrogen atom, as well as the atomic number for the n=1 state of all known atoms.

2. The holographic principle and the proton mass

The Bekenstein conjecture, first suggested by Jacob Bekenstein in the early 1970's, proposed that the entropy S or information contained in a given region of space, such as a black hole, is proportional to its surface horizon area **[18] [19] [20]**. Based on the laws of thermodynamics and the prediction of Hawking radiation, Hawking inferred and subsequently set the constant of proportionality to be ¼ of the surface horizon **[21]**. The Bekenstein-Hawking entropy of a black hole expressed in units of Planck area is thus given as,

$$S = \frac{A}{4\ell^2} \qquad \text{Eqn. 2}$$

where the Planck area, ℓ^2 is taken as one unit of entropy and A is the surface area of the black hole.

Bekenstein [22] further argued for the existence of a universal upper bound for the entropy of an arbitrary system with maximal radius r,

$$S \leq \frac{2\pi r E}{\hbar c} \qquad \text{Eqn. 3}$$

and found that this maximal bound is equivalent to the Bekenstein-Hawking entropy for a black hole (assuming $E = mc^2$). This confirmed the long suspected assumption that black holes have the maximum entropy for a given mass and size, which along with unitarity arguments led to the holographic principle of ‘t Hooft, where one bit of information is encoded by one Planck area [23] [24].

Following the holographic principle of ‘t hooft [24], based on the Bekenstein-Hawking formulae for the entropy of a black hole [25] [26], Haramein [15] [16] defines the holographic bit of information as an oscillating Planck spherical unit (PSU), given as

$$PSU = \frac{4}{3}\pi r_\ell^3 \qquad \text{Eqn. 4}$$

where $r_\ell = \frac{\ell}{2}$ and ℓ is the Planck length.

These PSUs, or Planck voxels, tile along the area of a spherical surface horizon, producing a holographic relationship with the interior information mass-energy density.

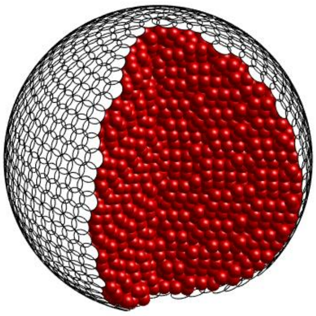

Figure 1: Schematic to illustrate the spherical Planck voxels packed within a spherical volume.

Theories of quantum gravity suggest that the quantum entropy of a black hole may not exactly equal A/4, as in reference [27]. In Haramein’s generalized holographic approach, he suggests that the information/entropy of a spherical surface horizon should be calculated in spherical bits and thus defines the surface information/entropy in terms of PSUs, such that,

$$\eta = \frac{A}{\pi r_\ell^2} \qquad \text{Eqn. 5}$$

where the Planck area, taken as one unit of information/entropy, is the equatorial disk of a Planck spherical unit, πr_ℓ^2 and A is the surface area of a spherical system. We note that in this definition, the entropy is slightly greater (~ 2.5 times) than that set by the Bekenstein Bound, and the proportionality constant is taken to be unity. To differentiate between models, the information/entropy S, encoded on the surface boundary in Haramein’s model is termed, $\eta \equiv S$.

As first proposed by ‘t Hooft the holographic principle states that the description of a volume of space can be encoded on its surface boundary, with one discrete degree of freedom per Planck area, which can be described as Boolean variables evolving with time [23]. Haramein, following his definition for surface information η, similarly defines the information/entropy within a volume of space in terms of PSU as,

$$R = \frac{V}{\frac{4}{3}\pi r_\ell^3} = \frac{r^3}{r_\ell^3} \qquad \text{Eqn. 6}$$

where V is the volume of the spherical entity and r is its radius.

In previous work [15] it was demonstrated that the holographic relationship between the transfer energy potential of the surface information and the volume information, equates to the gravitational mass of the system. It was thus found that for any black hole of Schwarzschild radius r_S the mass m_S can be given as,

$$m_S = \frac{R}{\eta} m_\ell \qquad \text{Eqn. 7}$$

where m_ℓ is the Planck mass, η is the number of PSU on the spherical surface horizon and R is the number of PSU within the spherical volume. Hence, a holographic gravitational mass equivalence to the Schwarzschild solution is obtained in terms of a discrete granular structure of spacetime at the Planck scale.

Furthermore, this inequality in energy potential between the surface information and the volume information, where $R > \eta$ for all $r > 2\ell$ suggests that both, the gravitational curvature potential is the result of an asymmetry in the information structure of spacetime, and the volume information is not only the result of the information/entropy surface bound of the local environment but may also be non-local, due to wormhole interactions as those proposed by the ER=EPR conjecture, where black hole interiors are connected through micro wormhole interactions [28].

Moreover, we find that the only radius at which the holographic ratio equals one (i.e. $R = \eta$), where all the volume information is encoded on the surface, is the condition

$$r_{S_\ell} = \frac{2Gm_\ell}{c^2} = 2\ell \qquad \text{Eqn. 8}$$

where r_{S_ℓ} is the Schwarzschild radius of a black hole with mass $m = m_\ell$.

In this case, the surface entropy η and the volume entropy R are thus calculated to be,

$$\eta_\ell = \frac{4\pi r_{S_\ell}^2}{\pi r_\ell^2} = \frac{4\pi(2\ell)^2}{\pi\left(\ell/2\right)^2} = 64 \qquad \text{Eqn. 9}$$

$$R_\ell = \frac{r_{s_\ell}^3}{r_\ell^3} = \frac{(2\ell)^3}{(\ell/2)^3} = 64$$

Eqn. 10

This results in a holographic ratio of, $\frac{R_\ell}{\eta_\ell} = 1$ yielding $m_\ell = \frac{R_\ell}{\eta_\ell} m_\ell$, such that a balanced state of equilibrium between the volume-to-surface information transfer potential is achieved, supporting the conjecture that due to its ultimate stability, the Planck entity is the fundamental granular kernel structure of spacetime forming a crystal-like structured lattice at the very fine scale of the quantum vacuum [29] [30].

Additionally, it is important to note that there is a factor of 2 or ½ between the Planck length and the Schwarzschild radius of a Planck mass black hole, and although its physical meaning has yet to be completely understood, it has been related to geometric considerations of motion, particle physics and cosmology, and commonly occurs in the most fundamental equations of physics [31]. However, the origin of this factor may be the result of the holographic surface-to-volume consideration of the fundamental geometric clustering of the structure of spacetime at the Planck scale, where one Planck mini black hole is a cluster bundle of Planck spherical vacuum oscillators [32].

Of course, these considerations lead to the exploration of the clustering of the structure of spacetime at the nucleonic scale. Here it was found that a precise value for the mass m_p and charge radius r_p of a proton can be given as,

$$m_p = 2\frac{\eta}{R} m_\ell = 2\phi m_\ell \quad \text{Eqn. 11}$$

$$r_p = 4\ell \frac{m_\ell}{m_p} = 0.841236(28) \times 10^{-13} cm \quad \text{Eqn. 12}$$

where ϕ is defined as a fundamental holographic ratio. Significantly, this value is within a 1σ agreement with the latest muonic measurements of the charge radius of the proton [15] [16], relative to a 7σ variance in the standard approach [17].

When utilizing this predicted radius (Eqn. 12), we then find that the gravitational coupling constant, $\alpha_g = F_g / F_s$, emerges as a natural consequence of the holographic ratio where,

$$\alpha_g = 4\phi^2 = 5.90595(24) \times 10^{-39} \quad \text{Eqn. 13}$$

again, finding the information surface-to-volume ratio related to the gravitational curvature mass potential, in this case, at the nucleonic confinement scale. We further evaluate the value of the velocity and mass dilation near or at the horizon of the proton and find it to be correlated to the range associated with the strong interaction defined by the Yukawa potential giving a first analytical solution to the confinement problem [15].

3. Determining the mass of the electron

In the previous section we described a generalized holographic solution which derives the proton mass from the granular Planck scale structure of spacetime in terms of a surface-to-volume information transfer potential. In an attempt to deepen our understanding, we consider the holographic ratio relationship as we extend the radius of the co-moving Planck particles to $r > r_p$.

In earlier work [15] it was shown that the confining force exerted by a proton follows Lorentz mass dilation and varies as a function of velocity. Following this approach, we evaluate the Lorentz transformation as a function of velocity and find the mass of the electron occurs at the standard expected velocity for the electron $v = \alpha c \approx \frac{c}{137}$ (see Figure 2), where α is the fine structure constant.

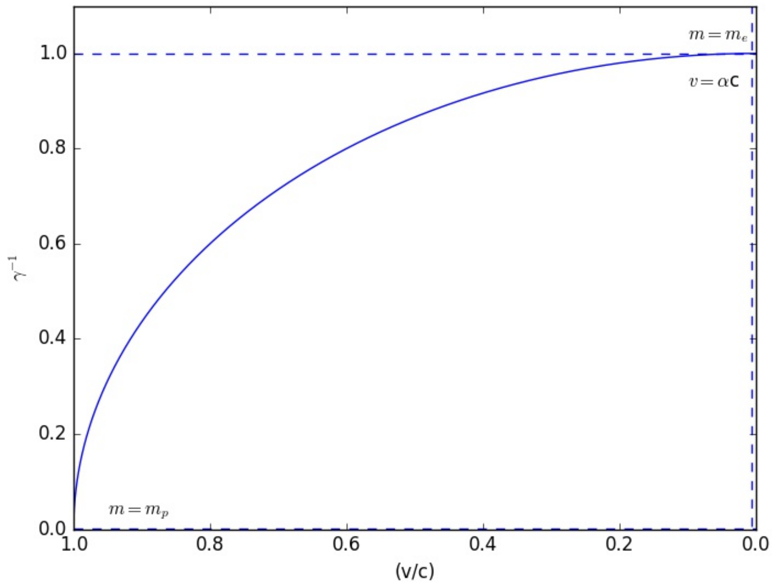

Figure 2: Graph showing the Lorentz factor $\gamma^{-1} = \sqrt{1-(v/c)^2}$ as a function of velocity for the proton to the electron.

It is thus reasonable to consider a velocity relationship in the holographic mass solution which becomes significant at $v < c$, and in this case appears at $r > r_p$. Thereafter, we evaluate this velocity relationship as we extend the radius, and thus the holographic surface-to-volume ratio, such that,

$$m_e = \beta \frac{v_\ell}{v_e} \phi_e m_\ell \qquad \text{Eqn. 14}$$

where β is a geometric parameter, v_ℓ is the velocity at the Planck scale, $v_e = \alpha c$ is the velocity of the electron, and ϕ_e is the holographic surface-to-volume ratio in terms of PSU.

With $\beta = 1/2$ (refer to previous section on the factor of 2 in physics), we find a mass in precise agreement with the experimental mass of the electron when the holographic ratio reaches $r = a_0$, where a_0 is the Bohr radius.

The solution for the mass of the electron (Eqn. 14) can thus be given as,

$$m_e = \frac{1}{2}\frac{v_\ell}{v_e}\frac{\eta_e}{R_e}m_\ell = \frac{1}{2}\frac{c}{\alpha c}\phi_e m_\ell = \frac{1}{2\alpha}\phi_e m_\ell \qquad \text{Eqn. 15}$$

where

$$\phi_e = \frac{\eta_e}{R_e} \text{ where } \eta_e = \frac{4\pi a_0^2}{\pi r_\ell^2} \text{ and } R_e = \frac{4/3\pi a_0^3}{4/3\pi r_\ell^3} = \frac{a_0^3}{r_\ell^3}$$

With this solution we find a mass of, $m_e = 9.10938(30)\times 10^{-28} g$, which compared to the measured CODATA 2014 value is accurate within 1σ and with a precision of 10^{-5} [1]. The precision and thus accuracy of our solution is restricted by the value of the Planck units which are dependent on experimental values given for the Gravitational constant, G. However, when the absolute value for the holographic mass solution for the electron is considered the mass is comparable with the experimental CODATA 2014 value to a greater degree of accuracy $<1\sigma$ and a precision of 10^{-8} with a confidence level of $99.9\dot{9}\%$.

This holographic mass solution, can as well be formulated in terms of charge relationships,

$$m_e = \frac{1}{2}\frac{v_\ell}{v_e}\phi_e m_\ell = \frac{1}{2\alpha}\phi_e m_\ell = \frac{1}{2}\frac{q_\ell^2}{q^2}\phi_e m_\ell \qquad \text{Eqn. 16}$$

where

$$\alpha = \frac{q^2}{q_\ell^2}$$

which when given in this form allows greater insight into the source of charge.

This solution, as well as being significantly accurate, gives us insight into the physical and mechanical dynamics of the granular Planck scale vacuum structure of spacetime and its role in the source of angular momentum, mass and charge. The definition clearly demonstrates that the differential angular velocities of the collective coherent behavior of Planck information bits determines specific scale boundary conditions and mass-energy relationships, analogous to the collective behavior of particles in a rotating fluid [33] or superfluid plasma [34].

This solution as well resolves the difficulty associated with hierarchy problems (we will address the electron-to-proton mass ratio below). The current quantum understanding resolves the hierarchy bare mass problem for the electron mass through the consideration of antimatter where positron and electron pairs pop in and out of the vacuum. These virtual particles smear out the charge over a greater radius such that the bare mass energy is cancelled by the electrostatic potential, where the greater the radius the lesser the need for fine tuning. In the solution presented here the electron is extended to a maximal radius of a_0 and we are able to demonstrate that the mass of the electron is a function of the Planck vacuum oscillators surface-to-volume holographic relationship, over this region of spacetime. The hierarchy bare mass problem is thus resolved by considering Planck vacuum oscillators acting coherently extending over a region of space equivalent to the Bohr hydrogen atom.

In much the same way that the electron analogy is proposed to resolve the Higgs hierarchy problem, with the inclusion of virtual supersymmetric particles, we could also assume that the surface to volume holographic relationship in the Higgs region of space would solve for the mass of the Higgs, where the Higgs radius would be of the order, $r_\ell < r_{Higgs} < r_p$.

The hierarchy problem associated with the mass of the electron and the mass of the proton can also be understood in terms of the surface-to-volume holographic ratio over their respective commoving regions of space, where the greater the radius the smaller the mass. The mass is thus a direct function of the commoving behaviour of the Planck vacuum, where the spin and mass decrease as a function of radius.

4. Extension of the holographic solution for radii less than the Bohr radius

When we further extend this solution for the n=1 state we find that, at radii $r < a_0$, the holographic mass solution increases as shown in Figure 3.

This application of the holographic solution gives us the following equation,

$$\frac{1}{2\alpha}\phi_r(r)m_\ell = Nm_e \qquad \text{Eqn. 17}$$

where $\phi_r(r)$ is the holographic ratio as a function of the radius r for $r < a_0$, and N is an integer.

With this solution, we could recognize N as being the atomic number Z, where for progressively smaller fractions of a_0 we find an interesting proportional relationship between the holographic mass and the mass of the electron (see Figure 3).

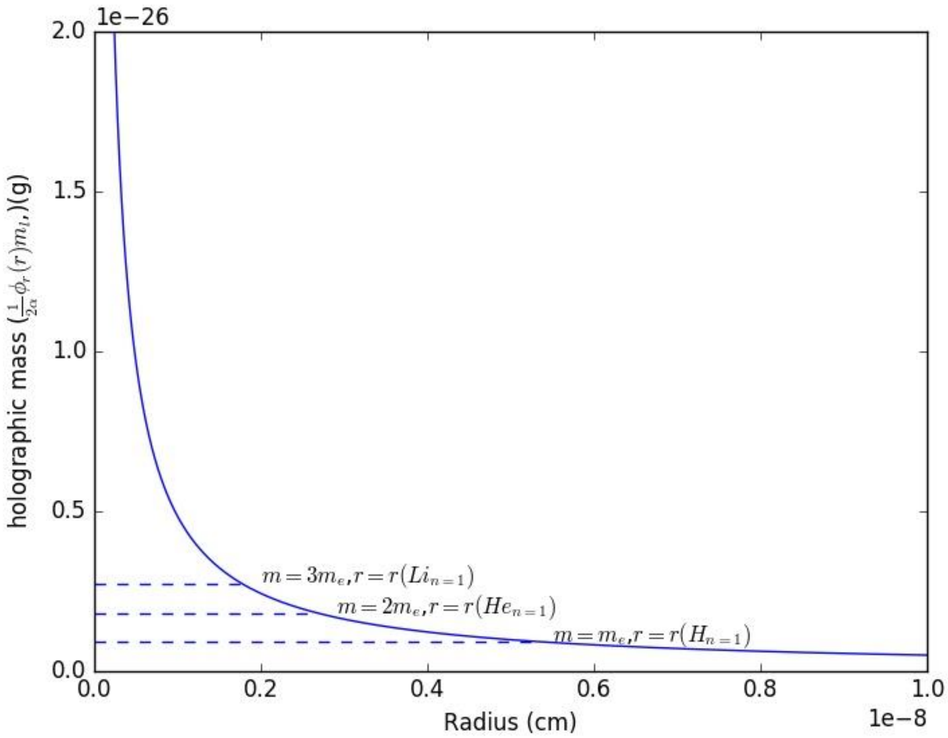

Figure 3: Graph to show the holographic mass solution as a function of radius. Note: the holographic mass is equal to m_e at corresponding radii of a_0/N. For example, the holographic mass: equals the mass of one electron at a radius of the hydrogen atom in its n=1 state; equals the mass of two electrons at a radius of the helium atom in its n=1 state; equals the mass of three electrons at a radius of the Lithium

atom in its n=1 state, and so on. Note this relationship is only shown on the graph for the first three elements, but continues for all known elements.

From this holographic mass solution, we are thus able to calculate the total mass of the electrons for all known elements, without the need for adding the atomic mass number, Z. We instead find that the atomic mass number Z could be a natural consequence of the holographic solution. As a result, a picture develops in which the structure of the Bohr atom and the charge and mass of both the proton and the electron are consequences of spin dynamics in the co-moving behaviour of the Planck scale granular structure of spacetime. This suggests that the confinement for the electron is a result of the quantum gravitational force exerted by the dynamics of the vacuum at the Planck scale. The electrostatic force can thus be accounted for in the same way the strong force is accounted for in the case of the proton [15] [16], where in both cases, the proton and the electron confinement is the result of a quantum force exerted through the granular Planck scale structure of spacetime.

5. Deriving the Rydberg constant, the fine structure constant and the proton to electron mass ratio

5.1 The Rydberg constant

The Rydberg constant is considered to be one of the most well-determined physical constants, with an accuracy of 7 parts to 10^{12} and is thus used to constrain the other physical constants [35]. However, as the same spectroscopic experiments are used to determine both the charge radius of the proton and the Rydberg constant, the recent muonic measurements of the charge radius of the proton implies that the Rydberg constant would change by $4-5\sigma$ [36] [37] [38]. This is known as the proton radius puzzle. The standard formula for the Rydberg constant is given as,

$$R_\infty = \frac{m_e \alpha^2 c}{2h} \qquad \text{Eqn. 18}$$

so any change in the experimentally determined value for the proton radius and thus the Rydberg constant will have a significant effect on the constraints defining the relationships between m_e, α, c and h.

In order to understand and subsequently infer any discrepancies between experimental values and theoretical values it is important to determine the underlying physical mechanism under which the Rydberg constant emerges. The holographic mass solution offers such a geometric mechanism providing a physical description and insight into how R emerges.

The standard formula for the mass of the electron (Eqn. 1), can be reduced to,

$$m_e = \frac{2R_\infty h}{c\alpha^2} = \frac{4\pi \ell m_\ell R_\infty}{\alpha^2} \qquad \text{Eqn. 19}$$

Equating this (Eqn. 19) with the geometric solution (Eqn. 15) gives,

$$m_e = \frac{4\pi\ell m_\ell R_\infty}{\alpha^2} = \frac{1}{2\alpha}\phi_e m_\ell$$

and thus

$$R_\infty = \frac{\alpha\phi_e}{8\pi\ell} = 1.097373(36)\times 10^5 cm^{-1} \qquad \text{Eqn. 20}$$

This definition offers a geometric solution for the Rydberg constant in agreement with the experimentally determined CODATA 2014 value.

As shown above, the holographic mass solution yields a correct value for both the charge radius of the proton, in agreement with the muonic radius measurement [15] [16] and the Rydberg constant independently, resolving the proton radius puzzle discrepancy from first principles.

5.2 The Fine Structure Constant and the Proton-to-Electron Mass Ratio

This approach can as well be extended to derive the fine structure constant, α and the proton to electron mass ratio, μ , in terms of ϕ_e .

If we equate the new geometric solution (Eqn. 20) with the standard definition (Eqn. 18) we get,

$$R_\infty = \frac{m_e\alpha^2 c}{2h} = \frac{\alpha\phi_e}{8\pi\ell}$$

and thus

$$\alpha = \frac{\phi_e h}{8\pi r_\ell m_e c} = \frac{\phi_e\lambda_e}{8\pi r_\ell} = 7.29735(34)\times 10^{-3} \qquad \text{Eqn. 21}$$

which is in agreement with that of the CODATA 2014 value. The ratio of the proton mass to the electron mass, μ can also be given in terms of the geometric solution (Eqn. 11 and Eqn. 15)

$$\mu = \frac{m_p}{m_e} = \frac{2\phi m_\ell}{\phi_e m_\ell / 2\alpha} = 4\alpha\frac{\phi}{\phi_e} = 1836.152(86) \qquad \text{Eqn.22}$$

Therefore, from the holographic ratio of Eqn. 22 we can now deduce a new expression for the proton radius r_p yielding a more precise prediction than the earlier result in reference [15] [16].

$$\mu = 4\alpha\frac{4r_\ell / r_p}{4r_\ell / a_0} = 4\alpha\frac{a_0}{r_p} \qquad \text{Eqn.23}$$

giving,

$$r_p = 4\frac{\alpha}{\mu}a_0 = 0.84123564042(46)\times 10^{-13} cm \qquad \text{Eqn. 24}$$

which agrees with the 2013 muonic measurement value of the proton charge radius $r_p = 0.84087(39) \times 10^{-13} cm$. More precise experimental values for the charge radius of the proton may confirm this theoretical prediction.

6. Summary

A new derivation for the mass of the electron is presented from first principles, where the mass is defined in terms of the holographic surface-to-volume ratio and the relationship of the electric charge at the Planck scale to that at the electron scale. This new derivation extends the holographic mass solution to the hydrogen Bohr atom and for all known elements, defining the atomic structure and charge as a consequence of the electromagnetic fluctuation of the Planck scale. Furthermore, the atomic number, Z emerges as a natural consequence of this geometric approach. The confinement for both the proton and the electron repulsive electrostatic force are now accounted for by a quantum gravitational force exerted by the granular Planck scale structure of spacetime. We conclude that this new approach offers an accurate value for the mass of the electron. As well, contrary to the standard calculation (as shown in Eqn. 1), it offers a physical understanding to the structure of spacetime at the quantum scale yielding significant insights in to the formation and source of the material world. Our results support our belief that such insights have significant value and should be developed further.

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